Topic Outline AP CALCULUS AB:

Unit 1: Basic tools and introduction to the derivative

A. Limits and properties of limits
   - Importance and novelty of limits
   - Traditional definitions of the limit
   - Graphical and numerical introduction to limits
   - Symbolic representation of limits
   - Properties of the limit
   - How to calculate limits using the TI 89

B. Continuity and why continuous functions are important
   - Traditional definition of continuity
   - Graphical understanding of continuity
   - How continuity relates to limits
   - Discontinuous Functions
   - Reason for studying continuous functions
   - Intermediate value theorem
   - Properties of continuous functions

C. Definition of the derivative and geometric interpretation of the derivative
   - Secant lines versus tangent lines
   - How to approximate tangent lines using secant lines
   - Definition of the derivative at a point
   - The different styles of notation for the derivative (and abuses thereof)
   - Graphical understanding of the derivative at a point
   - How the derivative relates to continuity and limits
   - Calculation of the derivative at a point for several functions
   - Attaching meaning to the symbols: Rates of Change
   - How to find the equation of a tangent line given the derivative

Unit 2: Calculations (of derivatives)

A. Derivative as a function v. derivative at a point
   - Learning to think of derivatives as functions by generalizing calculations in the previous unit.
   - Introduction to how the derivative relates graphically & conceptually to the original function

B. Review of power functions
   - Definition of power functions
   - Translations of power functions
   - Graphic review of power functions (end behavior, translation, roots…)

C. Derivative of power functions
   - Binomial theorem and Pascal's triangle
   - Proof of the power rule for derivatives
   - Calculations and visualizations of tangent lines
D. Review of exponent and logarithm functions
   - Definition of the general exponent functions using limits
   - Definition of logarithmic functions as inverses
   - Properties of each type of function graphically and algebraically

E. Derivative of exponent and logarithm functions
   - Limit definition of $e$
   - Exploration of special limits needed to calculate derivatives of logarithmic and exponential functions
   - Calculations of the derivatives of $a^x$ and $\log_a x$
   - Calculations and visualizations of tangent lines

F. Review of trigonometric functions
   - Unit circle
   - Graphs and definitions of trig functions
   - Relationship between the trig functions graphically, geometrically and algebraically

G. Derivative of trigonometric functions
   - Trigonometric identities and their geometric proofs
   - Geometric proofs of special limits needed to calculate the derivative of sine and cosine
   - Calculations of the derivative of sine and cosine
   - Calculations and visualizations of tangent lines

Unit 3: Derivative Rules

A. Constant, sum, product, quotient rules
   - Derivations of the constant, sum, product and quotient rules
   - Applications of these to find derivatives of functions like $\tan x$ ...

B. Chain rule
   - False proof of the chain rule
   - Discussion of the nuances needed for a complete proof
   - Importance of the chain rule
   - Practice using the chain rule

C. Derivatives of inverse functions
   - Review of inverse functions
   - Graphical interpretation of inverse functions
   - Definition of inverse functions
   - Use of chain rule to find the derivative of inverse functions
   - Calculation of the derivatives of inverse trig functions

D. Implicit differentiation
   - Definition and discussion of implicit functions
   - Discussion of the general method of finding the derivative of an implicitly defined function
   - Intro to related rates

E. How to use the TI-89 to differentiate functions
Unit 4: Graphing and the derivative

A. Local extrema versus global extrema
   - Graphical understanding of extrema
   - Numerical understanding of extrema
   - Fermat's Theorem and Proof
   - Extreme value theorem
   - Discussion of the contra positive and counterexamples
   - Closed interval method
   - Definition of critical points

C. Geometry of the first and second derivatives
   - Critical points
   - Definition of increasing/decreasing functions
   - First derivative test and proof
   - Graphical understanding of the first derivative and how it relates to the original function (calculations and visual checks using the graphing calculator)
   - Inflection points
   - Definition of concavity
   - Second derivative test
   - Graphical understanding of the second derivative and how it relates to the first derivative and the original function (calculations and visual checks using the graphing calculator)

C. Rolle's Theorem and Mean Value Theorem
   - Recalling important theorems from before
   - Visual description of Rolle's Theorem
   - Proof of Rolle's Theorem
   - Visual description of Mean Value Theorem
   - Proof of Mean Value Theorem
   - Basic types of questions (proving the uniqueness of a root ...)
   - Foreshadowing for Fundamental Theorem of Calculus

D. Limits at infinity
   - Description of asymptotes (horizontal, vertical, oblique) graphically and using limits
   - Calculation of limits involving infinity (including a review of long division and substitution in limits)
   - Discussion of relative growth rates of functions

E. Curve Sketching
   - Outline of the basic characteristics that a function may have
   - Examples of how to use the preceding techniques to graph functions without a graphing calculator
   - Practice graphing functions without calculator and checking the result with a calculator
Unit 4: Applications of derivatives

A. Velocity, acceleration and other rate of change problems
   - Relations between position, velocity and acceleration graphs
   - How derivatives relate to the above
   - Traditional falling body problem (Consider a ball thrown from an initial height, with an initial velocity. When does the ball reach the top of its trajectory? When is it not moving? When does the ball hit the ground, what speed is it going then? When is the speed the same as when it was thrown into the air? ...)

B. Optimization
   - Importance of critical points in applied mathematics
   - Graphical reinterpretation of extrema in the context of applied problems
   - Traditional optimization problems
   - Constraint equations

Related Rates
   - Review of implicit differentiation
   - Attaching meaning to symbols: Rates of change
   - Basic approach to solving related rate problems
   - Basic examples of related rate problems
   - Useful sources of formulas in solving related rate problems

Unit 5: Area Problem

A. Sigma notation
   - Sigma notation and properties of sigma notation
   - Traditional formulas for the sum of the first $n$ integers, squares and cubes.
   - Telescoping sums

B. Approximating area
   - Discussion of the term area and what it means "Has anyone seen a proof that the area of a circle is actually ..."
   - Known formulas for area
   - Approximation of the area under the graph of $f(x) = x^2$ above the interval $[0,1]$
   - Exact calculation of the area under the graph of $f(x) = x^2$ above the interval $[0,1]$

C. Riemann sums
   - Why limits are still useful
   - Definition of a Riemann sum
   - When you can find the area exactly
   - Approximation of areas using different sample points (Midpoint rule...)
   - Approximation of area using different geometric object (Trapezoidal rule ...)

D. Definite Integral
   - Notation and geometric meaning behind the notation
   - Exact calculations
   - Area function
   - Motivation for the FTC using the area function
Fundamental Theorem of Calculus part I
- Motivation and proof
- FTC part I and the chain rule

Fundamental Theorem of Calculus part II
- Recalling important theorems and formulas
- Motivation and proof
- Calculations (of area!) using FTC II

Indefinite Integrals and the Net Change Theorem
- Finding basic antiderivatives
- Calculating basic definite integrals using antiderivatives
- Traditional initial value problem for the height of falling object
- Displacement v. total distance traveled
- Graphical practice to determine total distance traveled and displacement

Unit 6: Differential equations

A. Intro to differential equations
- Definition of differential equation
- How differential equations fit into our previous framework
- Solving basic differential equations
- Initial conditions and the constant of integration
- Equilibrium solutions
- Introductions to population models

B. Slope fields
- Definition of the slope field
- How to use your calculator to generate slope fields
- Matching slope fields with the appropriate differential equation
- Finding equilibrium solutions graphically
- Integral curves and particular solutions
- How to approximate a solution curve using a slope field

C. Separable differential equations
- Definition of a separable differential equation
- General method for solving a separable differential equation
- Practice recognizing separable differential equations
- Traditional single brine tank problem

D. Exponential growth and decay
- Populations models
- Exponential model of populations
- Radioactive decay
- General solution of the differential equation associated with exponential growth
Unit 7: Applications

A. Substitution
- The chain rule "backwards"
- Substitution in definite integrals
- How to use the TI - 89 to calculate definite and indefinite integrals

B. Area between curves
- Graphical motivation for the formula for the area between two curves
- Nuances of the formula for the area between curves (when curves intersect...)

C. Volumes using revolution
- Practice revolving regions around different axes
- Motivation of calculating volumes using "slabs"
- Approximations of volume using slabs
- Relationship between volume and Riemann sums

D. Volumes of cross sections
- Volumes cross sectional shapes other than circles or washers

End of the year review

E. Volumes using shells
- Motivation for the calculation of volume using cylindrical shells
- Practice revolving regions around different axes
- Why we need two methods for calculating volume

F. Integration by Parts
G. Trig substitution for integrals
F. Logistics curves